TEMPERATURE DISTRIBUTION IN γ-RAY SHIELDING

D. P. Osanov

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Analytical expressions are obtained for the temperature distributions in γ ray shielding for isotropic and unidirectional beams with boundary conditions of the third kind. These are solved numerically for concrete shielding, and the effects of shield thickness, γ-ray scattering, boundary conditions and beam geometry on the temperature distribution profile are examined.

The release of heat inside shielding depends on the form and energy of the radiation, the shielding material and the attenuation functions, determined by the geometry of source and shield. For the majority of sources, the attenuation functions are now known in analytical form or have been tabulated in detail [1-3], and the heat release may be calculated with sufficient accuracy.

The heat release W(x) at depth x in a shield is related to the temperature T(x) by the equation of heat conduction, which, in the stationary one-dimensional case, has the form:

$$W(x) + k\frac{d^2T(x)}{dx^2} = 0.$$

The solution of this equation is presented in [4, 5, 6].

The temperature distribution in γ-ray shielding is determined below in a different way by solving the heat conduction equation for the most general boundary conditions of the third kind [7]. In this case, at the surface of the shield a relationship is specified between the unknown temperature and its normal derivative, which corresponds to free heat transfer to the surrounding medium:

$$\partial T/\partial n + h(T - T_m) = 0, \quad \text{where} \quad h = \alpha/k.$$

The temperature distribution in the shield is calculated for two cases: plane isotropic and plane unidirectional sources. It is considered that an isotropic beam of radiation falls on the shield from large extended sources with small self-absorption (surface and gas-filled sources), while the beam from volume sources with high self-absorption and from remote point sources is approximately unidirectional. It may be assumed with sufficient accuracy that the radiation from real γ ray sources is intermediate between that from plane isotropic and unidirectional sources. Thus, for identical heat fluxes, the temperature in the shield calculated for unidirectional irradiation may be taken as the upper limit, and that for an isotropic source as the lower limit for any real source.

Plane isotropic source. In the stationary case the heat conduction equation for an infinite plane isotropic source is written

$$S_1E_1(\mu x) + k\,d^2T/dx^2 = 0, \tag{1}$$

$$\frac{dT}{dx} = \frac{S_1}{k\mu}\Phi(\mu x) + C_1;$$

$$T(x) = \frac{S_1}{2k\mu^2}\mu x\left\{\Phi(\mu x) - \frac{1}{\mu x}\exp(-\mu x)\right\} + C_1x + C_2. \tag{2}$$

In these equations $\Phi(\mu x) = \exp(-\mu x) - \mu x E_1(\mu x)$.

To determine the constants of integration C_1 and C_2, boundary conditions of the third kind, corresponding to heat transfer from the surface of the shield to the surrounding medium, may be used. On the left at $x = 0$ $\frac{dT}{dx} - hT(0) = 0$, and on the right at $x = x_0$ $\frac{dT}{dx} + hT(x_0) = 0$. The temperature of the surrounding medium may be set equal to zero without loss of generality. After substituting the boundary conditions and transforming, we get

$$C_1 = \frac{S_1}{k\mu(2 + hx_0)}\left\{1 - \Phi(\mu x_0) - \frac{h\mu x_0}{2\mu}\left[\Phi(\mu x_0) - \frac{1}{\mu x_0}\exp(-\mu x_0) - 2 - \frac{h}{\mu} - \frac{1}{\mu x_0}\right]\right\} - \frac{S_1}{k\mu}\left(1 + \frac{h}{2\mu}\right);$$

$$C_2 = \frac{S_1}{k\mu(2h + h^2 x_0)}\left\{1 - \Phi(\mu x_0) - \frac{h\mu x_0}{2\mu} \times \right.$$
$$\left.\times\left[\Phi(\mu x_0) - \frac{1}{\mu x_0}\exp(-\mu x_0) - 2 - \frac{h}{\mu} - \frac{1}{\mu x_0}\right]\right\}.$$

The temperature at a depth x in the shield will then be given by

$$T(x) = \frac{S_1}{2k\mu^2}\left\{\mu x\left[\Phi(\mu x) - \frac{1}{\mu x}\exp(-\mu x) - 2 - \frac{h}{\mu}\right] + \right.$$
$$+ \frac{2}{h/\mu}\,\frac{h\mu x + \mu}{h\mu x_0 + 2\mu}\left[1 - \Phi(\mu x_0) - \frac{h}{2\mu}\mu x_0\left(\Phi(\mu x_0) - \right.\right.$$
$$\left.\left.\left. - \frac{1}{\mu x_0}\exp(-\mu x_0) - 2 - \frac{h}{\mu} - \frac{1}{\mu x_0}\right)\right]\right\} = \frac{S_1}{2k\mu} f(\mu x). \qquad (3)$$

This does not take into account γ-ray scattering in the shield. To do so, it is convenient to use the build-up factor expressed as a sum of two exponentials [8]

$$B(\mu x) = A\exp(\alpha_1 \mu x) + (1 + A)\exp(\alpha_2 \mu x).$$

In this case the heat release in the shield will equal $S_1[AE_1(\mu' x) + (1 - A)\times E_1(\mu'' x)]$, where $\mu' x = \mu x(1 + \alpha_1)$ and $\mu'' x = \mu x(1 + \alpha_2)$, and the expression for the temperature at depth x, taking account of γ-ray scattering, assumes the form

$$T(x) = \frac{S_1}{2k\mu}[Af(\mu' x) + (1 - A)f(\mu'' x)]. \qquad (4)$$

If in $dT/dx - hT(0) = 0$ and $dT/dx + hT(x_0) = 0$ we put $h \to \infty$ $(\alpha \to \infty)$, then the boundary conditions corresponding to heat transfer go over into the boundary conditions corresponding to a controlled temperature at the surfaces of the shield. Passing to the limit in (3), we get an expression for the temperature in the case when a temperature $T(0) = T(x_0) = 0$ is maintained at the inner and outer surfaces of the shield:

$$T(x) = \frac{S_1}{2k\mu^2}\mu x\left\{\Phi(\mu x) + \frac{1}{\mu x}[1 - \exp(-\mu x)] - \right.$$
$$\left. - \Phi(\mu x_0) - \frac{1}{\mu x_0}[1 - \exp(-\mu x_0)]\right\}. \qquad (5)$$

The equation for the temperature distribution in the case of controlled temperatures at the surfaces of the shield may also be obtained by direct solution of (2) for the boundary conditions $T(0)$ = const, $T(x_0)$ = const. Then, the constants of integration will be

$$C_1 = \frac{T(x_0) - T(0)}{x_0} - \frac{S_1}{2k\mu^2 x_0}\left\{1 + \mu x_0\left[\Phi(\mu x_0) - \frac{1}{\mu x_0}\exp(-\mu x_0)\right]\right\},$$

$$C_2 = T(0) + \frac{S_1}{2k\mu^2}.$$

Putting these values in (2) and writing $T(0) - T(x_0) = \Delta T$, we get

$$T(x) = T_0 - \Delta T\frac{x}{x_0} + \frac{S_1}{2k\mu^2}\mu x\left\{\Phi(\mu x) + \frac{1}{\mu x}[1 - \exp(-\mu x)] - \right.$$
$$\left. - \Phi(\mu x_0) - \frac{1}{\mu x_0}[1 - \exp(-\mu x_0)]\right\}. \qquad (6)$$

For $T(0) = T(x_0) = 0$, (6) is identical with (5).

Plane unidirectional source. The heat release in a shield at depth x due to γ-radiation from an infinite plane unidirectional source is $S_2\exp(-\mu x)$. In this case, $S_2 = N_0 E\gamma C_2$, where $C_2 = 15\cdot 10^{-14}$ joules/Mev.

The stationary heat conduction equation for this source has the form

$$S_2 \exp(-\mu x) + k\, d^2T/dX^2 = 0.$$

Integrating, we get

$$dT/dx = S^2/k\mu \exp(-\mu x) + C_1$$

and

$$T(x) = -\frac{S_2}{k\mu^2} \exp(-\mu x) + C_1 x + C_2. \tag{7}$$

The constants of integration C_1 and C_2, determined using boundary conditions of the third kind, are respectively

$$C_1 = \frac{S_2}{k\mu(2+x_0)} \left\{ 1 - \exp(-\mu x_0) + \frac{h}{\mu} \times \right.$$

$$\left. \times \left[1 + \exp(-\mu x_0) + \mu x_0 \left(1 + \frac{h}{\mu}\right)\right]\right\} - \frac{S_2}{k\mu}\left(1 + \frac{h}{\mu}\right),$$

$$C_2 = \frac{S_2}{k\mu(2h + h^2 x_0)} \left\{ 1 - \exp(-\mu x_0) + \right.$$

$$\left. + \frac{h}{\mu}\left[1 + \exp(-\mu x_0) + \mu x_0 \left(1 + \frac{h}{\mu}\right)\right]\right\}.$$

Thus, for the temperature distribution in the shield due to unidirectional radiation we have

$$T(x) = \frac{S_2}{k\mu^2} \left\{ \frac{\mu x + \mu/h}{2 + hx_0} \left[1 - \exp(-\mu x_0) + \right.\right.$$

$$\left. + \frac{h}{\mu}(1 + \exp(-\mu x_0) + \mu x_0 + hx_0)\right] -$$

$$\left. - \exp(-\mu x) - \mu x \left(1 + \frac{h}{\mu}\right)\right\} = \frac{S_2}{k\mu^2} \varphi(\mu x). \tag{8}$$

Setting $h \to \infty$ in (8), we can go over to the solution for boundary conditions of the first kind, when $T(0) = T(x_0) = 0$

$$T(x) = \frac{S_2}{k\mu^2} \left\{ 1 - \exp(-\mu x) - \frac{x}{x_0}[1 - \exp(-\mu x_0)]\right\}. \tag{9}$$

If the boundary conditions are in the form $T(0) = \text{const.}$, $T(x_0) = \text{const.}$, then the temperature at depth x in the shield is given by

$$T(x) = T(0) - \Delta T \frac{x}{x_0} + \frac{S_2}{k\mu^2} \times$$

$$\times \left\{ 1 - \exp(-\mu x) - \frac{x}{x_0}[1 - \exp(-\mu x_0)]\right\}. \tag{10}$$

It is evident that for $T(0) = T(x_0) = 0$ this is the same as (9).

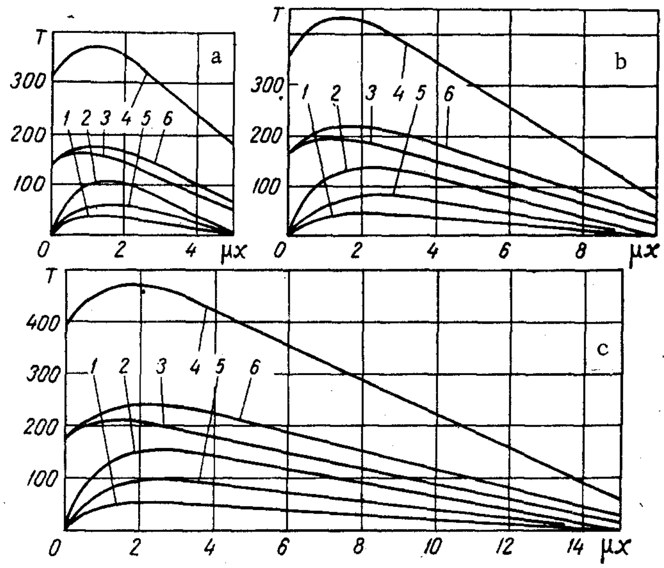
To take account of the effect of scattered γ-radiation from a unidirectional source on the temperature distribution in the shield, the method used above for a plane isotropic source may be employed.

Calculation of temperature distributions in concrete shielding. Concrete has good mechanical and physical characteristics combined with comparatively low cost and is thus one of the shielding materials most frequently used. However, concrete as a shielding material has a serious deficiency – its low thermal conductivity. Hence, the dissipation of heat generated within a concrete shield by absorption of radiation energy is a difficult problem.

Owing to unequal heating considerable temperature stresses may develop in the concrete [9], to combat which it is necessary to add reinforcement or provide expansion joints.

The need for and degree of reinforcement or other means of relieving the temperature stresses can only be established from the temperature curves for the concrete shield. Thus, for a temperature difference over the thickness of the shield of 40-50°, only the normal reinforcement is required, but for a temperature drop of 90° and above, extra reinforcement must be added.

The temperature distribution in a concrete shield (see figure) was calculated for isotropic and unidirectional γ-ray beams. Curve 1 is calculated for an isotropic beam without allowance for γ-ray scattering in the concrete and for bound-



Temperature distribution inside concrete shield (gravel concrete) of thickness 5 (a), 10 (b), and 15 (c) γ-quanta mean free paths for $q_0 = 0.42 \cdot 10^4$ watt·m^{-2} and γ-ray energy E = = 1.25 MeV (Co^{60}), k = 1.45 watt·m^{-1} per unit temperature gradient, $T_m = 0°C$, $\alpha = 11.7$ watt·m^{-2} per unit temperature gradient [11], $\mu = 0.126$ cm^{-1} [8].

ary conditions of the first kind (5); curve 2 differs from curve 1 in that the calculation includes multiple γ-ray scattering in the concrete using values for the coefficients A, α_1, and α_2 from [8]; curve 3 is calculated for an isotropic beam without allowance for γ-ray scattering and for boundary conditions of the third kind; curve 4 is for the same case, but includes the effect of γ ray scattering; curve 5 is for unidirectional radiation without allowance for scattering in the shield and for boundary conditions of the first kind (9); curve 6 gives the temperature distribution due to an unscattered unidirectional beam under conditions of free heat transfer between the shield and the surrounding medium (8).

The shape of all the temperature distribution curves is approximately the same; they are asymmetric with respect to the center of the shield and have a more or less clearly defined maximum of variable height and position. It may be noted that the temperature curves shown in [5, 6] have the same shape. However, the location of the temperature maximum, established on the basis of approximate calculations in [5, 6] (x ≈ 30 cm from inner surface of the shield) in no case agrees with the locations given by exact equations of the form dT/dx = 0. In fact, the temperature maximum is located no more than 10-15 cm from the surface of the concrete.

Let us now consider the effect of the individual factors on the temperature distribution curves for concrete shielding.

Effect of boundary conditions. Boundary conditions have a substantial effect on the temperature distribution inside the shield. Comparing curves 1 and 3, 2 and 4, and 5 and 6, we note that for free heat transfer at the shield surface, the maximum is located considerably closer to the surface (about 30-50%) than under constant temperature conditions. In addition, at the same temperature of the external medium, the height of the temperature maximum for free heat transfer is more than three times greater than the maximum for constant surface temperature for an isotropic beam, and 2-2.5 times greater for a unidirectional beam.

Effect of γ-ray scattering in shield. γ-Ray scattering in the concrete shield leads in all cases to an increase in temperature over that due to an unscattered beam. The difference between the curves for the case of free heat transfer increases with shield thickness. When allowance is made for scattering, the temperature maximum is displaced 50-100% into the shield and its height, for an isotropic beam, is increased 2.5-3 times. The temperature distribution due to a unidirectional beam with allowance for γ-ray scattering has not been calculated for lack of an analytical dependence of build-up factor on shield thickness for this geometry. It may, however, be shown that since in absolute terms the build-up factor for a plane unidirectional source is always less than that for a plane isotropic source, the temperature difference between these sources with allowance for scattering must be less than the difference between the corresponding distributions without such allowance (curves 1-5 and 3-6).

Effect of geometry of γ-ray beam. It may be seen from the figure that for the same heat flux at the surface the temperature inside the shield and the depth at which the temperature maximum occurs are greater for unidirectional than for isotropic irradiation. This is obviously related to the greater penetrating power of the unidirectional beam. It should also be noted that the effect of beam geometry on the temperature curve is considerably weaker than the effects of the boundary conditions at the surface of the shield and γ ray scattering. This fact may be used in rough estimates of shield heating without account for the actual source geometry, using, for example, the equations and curves for unidirectional irradiation.

Effect of shield thickness. The conditions for dissipation of heat from the inner part of the shield deteriorate with increase in thickness; thus, all the curves are displaced in the direction of increased temperature. At the same time, the temperature maximum is displaced into the shield by a distance $\mu x \approx 1\text{-}1.5$.

The results obtained permit the evaluation of approximate practical recommendations on safe conditions of irradiation for concrete. For example, in [12] Lane gives a limiting (in the sense of temperature limitations) value for the energy flux to which concrete may be exposed. This is $2 \cdot 10^{11}$ MeV $\cdot$ cm^{-2} $\cdot$ sec^{-1}, which corresponds to $3 \cdot 10^2$ watt $\times$ $\times$ m^{-2}. It is also shown that in this case the concrete temperature rises by 28°C. Obviously, for $0.4 \cdot 10^4$ watt $\cdot$ m^{-2} the temperature rise should be $\approx$360°C.

For comparison, suppose the temperature distribution due to an isotropic beam with allowance for γ-ray scattering and free heat transfer at the shield edges is used (curve 4). Apparently, this is one of the typical cases often met with in practice. The maximum temperature on curves 4 is $\approx$380°C for $\mu x_0 = 5$, 440° for $\mu x_0 = 10$, and 475° for $\mu x_0 = 15$. These values agree to within an order of magnitude with Lane's estimate [12], but the difference is, nevertheless, substantial, especially for a thick shield. However, Lane did not specify the shield thickness or the conditions of heat removal to which his limit of $2 \cdot 10^{11}$ MeV $\cdot$ cm^{-2} $\cdot$ sec^{-1} corresponds, or how this value may change depending on shield thickness and other conditions.

The equations and temperature curves for concrete presented in this paper may be useful in estimating the possible heating of protective shielding built around powerful γ-ray sources. Since the dependence between the temperature inside the shielding T(x) and the heat flux at the surface q is linear (assuming that radiation losses are negligible), T(x) may be determined for a given q from the relation $T(x) = T_0(x)q/q_0$, where $T_0(x)$ is the temperature at depth x in the concrete for a heat flux $q_0 = 0.42 \cdot 10^4$ watt $\cdot$ m^{-2}, a value determined from the graphs in the figure.

NOTATION

α and k – heat transfer coefficient and thermal conductivity; $S_1E_1(\mu x)$ – heat release at depth x in shield; $E_1(\mu x)$ – integral exponential function; μ – attenuation factor for γ-radiation; S_1 – specific heat release in shield due to unattenuated γ ray beam for a plane isotropic source; σ – density of surface activity; n – yield of γ-quanta per decay; E – energy of γ-quanta; γ – coefficient of energy absorption in shield material; N_0 – number of γ-quanta normally incident on unit area of shield surface; q_0 – heat flux to inner surface of shield.

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Biophysical Institute, Moscow